Approximate Estimation of Distributed Networked Control Systems

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***Abstract*— In this paper, we present two approximate filtering algorithms for estimating states of a distributed networked control system (DNCS). A DNCS consists of multiple agents communicating over a lossy communication channel, *e.g.*, wireless channel. While the time complexity of the exact method can be exponential in the number of communication links, the time complexity of an approximate method is not dependent on the number of communication links. In addition, we discuss the general conditions for stable state estimation of DNCSs.**

I. INTRODUCTION

With the recent developments in communication, computing, and control systems, a networked control system (NCS) has received a fair amount of attention recently. In a general sense, an NCS consists of spatially distributed multiple systems or agents equipped with sensors, actuators, and computing and communication devices. The operation of each agent is coordinated over a communication network. The examples of an NCS include sensor networks [1], [2], networked autonomous mobile agents [3], *e.g.*, a team of UAVs, and arrays of micro or micro-electromechanical sensors (MEMS) devices.

Recently, different aspects of NCSs have been studied extensively. Sinopoli *et al.* [4] showed the phase transition behavior of the Kalman filter when the measurement packet loss is modeled by a Bernoulli random process and established the relationship between the speed of dynamics and the packet loss rate for stable state estimation of the system. Similar estimation problems are discussed in [5], [6], [7]. The control problems over an unreliable communication channel have been studied by many authors, including [8], [9], [10]. The stability of NCSs has been also studied in [11], [12].

There is a growing interest in consensus and coordination of networked systems inspired by the model by Vicsek *et al.* [13], in which a large number of particles (or autonomous agents) move at a constant speed but with different headings. At each discrete time, each particle updates its heading based on the average heading of its neighboring particles. The analysis of the Vicsek model in different forms are reported in [14], [15], [16].

In literature, a single plant is usually assumed for an NCS and the links between the plant and the estimator or controller are closed by a common (unreliable) communication channel. This notion is extended by a distributed networked control system (DNCS) in which there are multiple agents communicating over a lossy communication channel [17]. A DNCS extends an NCS to model a distributed multi-agent system such as the Vicsek model. The best examples of such system include ad-hoc wireless sensor networks and a network of mobile agents. The exact state estimation method based on the Kalman filter is introduced in [17]. However, the time complexity of the exact method can be exponential in the number of communication links. In this paper, we address this issue by developing two approximate filtering algorithms for estimating states of a DNCS. The approximate filtering algorithms bound the state estimation error of the exact filtering algorithm and the time complexity of approximate methods is not dependent on the number of communication links.

The stability of estimators under a lossy communication channel is studied in [4], [5]. However, the extension of the result to the general case with an arbitrary number of lossy communication links is unknown. While computing the exact communication link probabilities required for stable state estimation is nontrivial, we describe the general conditions for stable state estimation using jump linear system theory.

The remainder of this paper is structured as follows. The dynamic models of DNCSs are described in Section II. The modified Kalman filtering methods for DNCSs are described in Section III and the approximate filtering algorithms are derived in Section IV. Conditions for stable state estimation is discussed in Section V and simulation results are described in Section VI.

II. DISTRIBUTED NETWORKED CONTROL SYSTEMS WITH LOSSY LINKS

Let us first consider a distributed control system consisting of N agents, in which there is no communication loss. The discrete-time linear dynamic model of the agent j can be described as following:

$$x_j(k+1) = \sum_{i=1}^{N} A_{ij} x_i(k) + G_j w_j(k) \qquad (1)$$

where $k \in \mathbb{Z}^+$, $x_j(k) \in \mathbb{R}^{n_x}$ is the state of the agent j at time k, $w_j(k) \in \mathbb{R}^{n_w}$ is a white noise process, $A_{ij} \in \mathbb{R}^{n_x \times n_x}$, and $G_j \in \mathbb{R}^{n_x \times n_w}$. Hence, the state of the agent j is governed by the previous states of all N agents. We can also consider $A_{ij} x_i(k)$ as a control input from the agent i to the agent j for $i \neq j$.

Now consider a distributed networked control system (DNCS), in which agents communicate with each other over

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a lossy communication channel, *e.g.*, wireless channel. We assume an erasure channel between a pair of agents. At each time k, a packet sent by the agent i is correctly received by the agent j with probability p_{ij}. We form a communication matrix $P_{\text{com}} = [p_{ij}]$. Let $Z_{ij}(k) \in \{0,1\}$ be a Bernoulli random variable, such that $Z_{ij}(k) = 1$ if a packet sent by the agent i is correctly received by the agent j at time k, otherwise, $Z_{ij}(k) = 0$. Since there is no communication loss within an agent, $p_{ii} = 1$ and $Z_{ii}(k) = 1$ for all i and k. For each (i,j) pair, $\{Z_{ij}(k)\}$ are i.i.d. (independent identically distributed) random variables such that $P(Z_{ij}(k) = 1) = p_{ij}$ for all k; and $Z_{ij}(k)$ are independent from $Z_{lm}(k)$ for $l \neq i$ or $m \neq j$. Then we can write the dynamic model of the agent j under lossy links as following:

$$x_j(k+1) = \sum_{i=1}^{N} Z_{ij}(k) A_{ij} x_i(k) + G_j w_j(k). \qquad (2)$$

Let $x(k) = [x_1(k)^T, \dots, x_N(k)^T]^T$ and $w(k) = [w_1(k)^T, \dots, w_N(k)^T]^T$, where y^T is a transpose of y. Let $\bar{A}_{ij}$ be a $Nn_x \times Nn_x$ block matrix. The entries of $\bar{A}_{ij}$ are all zeroes except the (j,i)-th block is A_{ij}. For example, when $N = 2$

$$\bar{A}_{12} = \begin{bmatrix} \mathbf{0}_{n_x} & \mathbf{0}_{n_x} \\ A_{12} & \mathbf{0}_{n_x} \end{bmatrix},$$

where $\mathbf{0}_{n_x}$ is a $n_x \times n_x$ zero matrix. Then the discrete-time linear dynamic model of the DNCS with lossy links can be represented as following:

$$x(k+1) = \left(\sum_{i=1}^{N} \sum_{j=1}^{N} Z_{ij}(k) \bar{A}_{ij} \right) x(k) + G w(k), \qquad (3)$$

where G is a block diagonal matrix of $G_1, \dots, G_N$.

For notational convenience, we introduce a new index $n \in \{1, \dots, N^2\}$ such that ij is indexed by $n = N(i-1) + j$. With this new index n, the dynamic model (3) can be rewritten as

$$x(k+1) = \left(\sum_{n=1}^{N^2} Z_n(k) \bar{A}_n \right) x(k) + G w(k). \qquad (4)$$

By letting $A(k) = \left(\sum_{n=1}^{N^2} Z_n(k) \bar{A}_n \right)$, we see that (4) is a time-varying linear dynamic model:

$$x(k+1) = A(k) x(k) + G w(k). \qquad (5)$$

Until now we have assumed that $\bar{A}_n$ is fixed for each n. Now suppose a more general case where the matrix A is time-varying and its values are determined by the communication link configuration $Z(k) = [Z_1(k), \dots, Z_{N^2}(k)]^T$. Hence, A is a function of $Z(k)$ and this general case can be described as

$$x(k+1) = A(Z(k)) x(k) + G w(k). \qquad (6)$$

The dynamic model (6) or (4) is a special case of the linear hybrid model or a jump linear system [18] since $A(k)$ takes an element from a set of a finite number of matrices. We will call the dynamic model (4) as the "simple" DNCS dynamic model and (6) as the "general" DNCS dynamic model.

III. Modified Kalman Filter for DNCSs

In this section, we outline a recursive filtering algorithm described in [17] for the general DNCS dynamic model (6). Since $Z(k)$ is independent from $Z(t)$ for $t \neq k$, we derive an optimal linear filter. Notice that we denote $Z(k)$ by Z when there is no confusion.

Consider the general DNCS dynamic model (6), where $w(k)$ is a Gaussian noise with zero mean and covariance Q, and the following measurement model:

$$y(k) = C x(k) + v(k), \qquad (7)$$

where $y(k) \in \mathbb{R}^{n_y}$ is a measurement at time k, $C \in \mathbb{R}^{n_y \times N n_x}$, and $v(k)$ is a Gaussian noise with zero mean and covariance R. Hence, we are assuming that the measurements are collected by a remote sensor or by a sensor in one of the agents. Notice that $Z(k)$ is not observed.

The following terms are defined to describe the modified Kalman filter.

$$\begin{aligned} \hat{x}(k|k) &:= \mathbb{E}\left[x(k) | \mathbf{y_k}\right] \\ P(k|k) &:= \mathbb{E}\left[e(k)e(k)^T | \mathbf{y_k}\right] \\ \hat{x}(k+1|k) &:= \mathbb{E}\left[x(k+1) | \mathbf{y_k}\right] \\ P(k+1|k) &:= \mathbb{E}\left[e(k+1|k)e(k+1|k)^T | \mathbf{y_k}\right], \end{aligned}$$

where $\mathbf{y_k} = \{y(t) : 0 \le t \le k\}$, $e(k|k) = x(k) - \hat{x}(k|k)$, and $e(k+1|k) = x(k+1) - \hat{x}(k+1|k)$.

Suppose that we have estimates $\hat{x}(k|k)$ and $P(k|k)$ from time k. At time $k+1$, a new measurement $y(k+1)$ is received and our goal is to estimate $\hat{x}(k+1|k+1)$ and $P(k+1|k+1)$ from $\hat{x}(k|k)$, $P(k|k)$, and $y(k+1)$. First, we compute $\hat{x}(k+1|k)$ and $P(k+1|k)$.

$$\begin{aligned} \hat{x}(k+1|k) &= \mathbb{E}\left[x(k+1) | \mathbf{y_k}\right] \\ &= \mathbb{E}\left[A(Z)x(k) + Gw(k) | \mathbf{y_k}\right] \\ &= \hat{A}\hat{x}(k|k), \end{aligned} \qquad (8)$$

where

$$\hat{A} = \sum_{z \in \mathcal{Z}} p_z A(z)$$

is the expected value of $A(Z)$. Here, $p_z = P(Z = z)$, and $\mathcal{Z}$ is a set of all possible outcome vectors for Z, *i.e.*, $\mathcal{Z}$ is a set of all possible communication link configurations.

The prediction covariance can be computed as following.

$$\begin{aligned} P(k+1|k) &= \mathbb{E}[e(k+1|k)e(k+1|k)^T | \mathbf{y_k}] \\ &= GQG^T + \textstyle\sum_{z \in \mathcal{Z}} p_z A(z) P(k|k) A(z)^T \\ &\quad + \textstyle\sum_{z \in \mathcal{Z}} p_z A(z) \hat{x}(k|k) \hat{x}(k|k)^T (A(z) - \hat{A})^T. \end{aligned} \qquad (9)$$

Given $\hat{x}(k+1|k)$ and $P(k+1|k)$, $\hat{x}(k+1|k+1)$ and $P(k+1|k+1)$ are computed as in the standard Kalman filter.

$$\begin{aligned} \hat{x}(k+1|k+1) &= \hat{x}(k+1|k) \qquad (10) \\ &+ K(k+1)(y(k+1) - C\hat{x}(k+1|k)) \\ P(k+1|k+1) &= P(k+1|k) \qquad (11) \\ &- K(k+1) C P(k+1|k), \end{aligned}$$

where $K(k+1) = P(k+1|k) C^T (C P(k+1|k) C^T + R)^{-1}$.

IV. Approximate Kalman Filter for DNCSs

The modified KF proposed in Section III for the general DNCS is an optimal linear filter but the time complexity of the algorithm can be exponential in N since the size of $\mathcal{Z}$ is $O(2^{N(N-1)})$ in the worst case, *i.e.*, when all agents can communicate with each other. In this section, we describe two approximate Kalman filtering methods for the general DNCS dynamic model (6) which are more computationally efficient than the modified KF by avoiding the enumeration over $\mathcal{Z}$. Since the computation of $P(k+1|k)$ is the only time-consuming process, we propose two filtering method which can bound $P(k+1|k)$. We use the notation $A \succ 0$ if A is a positive definite matrix and $A \succeq 0$ if A is a positive semidefinite matrix.

A. Lower-bound KF for General DNCS

The *lower-bound KF* (lb-KF) is the same as the modified KF described in Section III, except we approximate $P(k+1|k)$ by $\underline{P}(k+1|k)$ and $P(k|k)$ by $\underline{P}(k|k)$. The covariances are updated as following:

$$\begin{aligned} \underline{P}(k+1|k) &= \hat{A}\underline{P}(k|k)\hat{A}^T + GQG^T \quad (12) \\ \underline{P}(k+1|k+1) &= \underline{P}(k+1|k) \quad (13) \\ &- \underline{K}(k+1)C\underline{P}(k+1|k), \end{aligned}$$

where $\hat{A}$ is the expected value of $A(Z)$ and $\underline{K}(k+1) = \underline{P}(k+1|k)C^T(C\underline{P}(k+1|k)C^T+R)^{-1}$. Notice that $\hat{A}$ can be computed in advance and the lb-KF avoids the enumeration over $\mathcal{Z}$.

Lemma 1: If $\underline{P}(k|k) \preceq P(k|k)$, then $\underline{P}(k+1|k) \preceq P(k+1|k)$.

Proof: Using (9), we have

$$\begin{aligned} &P(k+1|k) - \underline{P}(k+1|k) \\ &= \mathbb{E}[A(Z)P(k|k)A(Z)^T] + \mathbb{E}[A(Z)\hat{x}(k|k)\hat{x}(k|k)^T A(Z)^T] \\ &\quad -\hat{A}\hat{x}(k|k)\hat{x}(k|k)^T\hat{A}^T - \hat{A}\underline{P}(k|k)\hat{A}^T \\ &= P_1 + P_2, \end{aligned}$$

where $P_1 = \mathbb{E}[A(Z)P(k|k)A(Z)^T] - \hat{A}\underline{P}(k|k)\hat{A}^T$ and $P_2 = \mathbb{E}[A(Z)\hat{x}(k|k)\hat{x}(k|k)^T A(Z)] - \hat{A}\hat{x}(k|k)\hat{x}(k|k)^T\hat{A}^T$.

If $P_1 \succeq 0$ and $P_2 \succeq 0$, then $P(k+1|k) - \underline{P}(k+1|k) \succeq 0$ and it completes the proof.

$$\begin{aligned} P_1 &= \mathbb{E}[A(Z)P(k|k)A(Z)^T] - \hat{A}\underline{P}(k|k)\hat{A}^T \\ &- \hat{A}P(k|k)\hat{A}^T + \hat{A}P(k|k)\hat{A}^T \\ &= \mathbb{E}[A(Z)P(k|k)A(Z)^T] - \hat{A}P(k|k)\hat{A}^T \\ &+ \hat{A}(P(k|k) - \underline{P}(k|k))\hat{A}^T. \end{aligned}$$

Since $P(k|k)$ is a symmetric matrix, $P(k|k)$ can be decomposed into $P(k|k) = U_1 D_1 U_1^T$, where U_1 is a unitary matrix and D_1 is a diagonal matrix. Hence,

$$\begin{aligned} P_1 &= \mathbb{E}[(A(Z)U_1D_1^{1/2})(A(Z)U_1D_1^{1/2})^T] \\ &- \mathbb{E}[A(Z)U_1D_1^{1/2}]\mathbb{E}[A(Z)U_1D_1^{1/2}]^T \\ &+ \hat{A}(P(k|k) - \underline{P}(k|k))\hat{A}^T \\ &= \mathrm{Cov}[A(Z)U_1D_1^{1/2}] + \hat{A}(P(k|k) - \underline{P}(k|k))\hat{A}^T, \end{aligned}$$

where $\mathrm{Cov}[H]$ denotes the covariance matrix of H. Since a covariance matrix is positive definite and $P(k|k) - \underline{P}(k|k) \succeq 0$ by assumption, $P_1 \succeq 0$. P_2 is a covariance matrix since $\hat{x}(k|k)\hat{x}(k|k)^T$ is symmetric, hence $P_2 \succeq 0$. ■

Lemma 2: If $\underline{P}(k+1|k) \preceq P(k+1|k)$, then $\underline{P}(k+1|k+1) \preceq P(k+1|k+1)$.

Proof: Applying the matrix inversion lemma to (11), we have $P(k+1|k+1) = \left(P(k+1|k)^{-1} + C^T R^{-1} C\right)^{-1}$. Let $P = P(k+1|k)$ and $\underline{P} = \underline{P}(k+1|k)$. Then

$$\begin{aligned} P &\succeq \underline{P} \\ P^{-1} &\preceq \underline{P}^{-1} \\ P^{-1} + C^T R^{-1} C &\preceq \underline{P}^{-1} + C^T R^{-1} C \\ \left(P^{-1} + C^T R^{-1} C\right)^{-1} &\succeq \left(\underline{P}^{-1} + C^T R^{-1} C\right)^{-1} \\ P(k+1|k+1) &\succeq \underline{P}(k+1|k+1). \end{aligned}$$

■

Finally, using Lemma 1, Lemma 2, and the induction hypothesis, we have the following theorem showing that the lb-KF maintains the state error covariance which is upper-bounded by the state error covariance of the modified KF.

Theorem 1: If the lb-KF starts with an initial covariance $\underline{P}(0|0)$, such that $\underline{P}(0|0) \preceq P(0|0)$, then $\underline{P}(k|k) \preceq P(k|k)$ for all $k \geq 0$.

B. Upper-bound KF for General DNCS

Similar to the lb-KF, the *upper-bound KF* (ub-KF) approximates $P(k+1|k)$ by $\bar{P}(k+1|k)$ and $P(k|k)$ by $\bar{P}(k|k)$. Let $\lambda_{\max} = \lambda_{\max}(\bar{P}(k|k)) + \lambda_{\max}(\hat{x}(k|k)\hat{x}(k|k)^T)$, where $\lambda_{\max}(S)$ denotes the maximum eigenvalue of S. The covariances are updated as following:

$$\begin{aligned} \bar{P}(k+1|k) &= \lambda_{\max}\mathbb{E}[A(Z)A(Z)^T] \quad (14) \\ &- \hat{A}\bar{x}(k|k)\bar{x}(k|k)^T\hat{A}^T + GQG^T \\ \bar{P}(k+1|k+1) &= \bar{P}(k+1|k) \quad (15) \\ &- \bar{K}(k+1)C\bar{P}(k+1|k), \end{aligned}$$

where $\hat{A}$ is the expected value of $A(Z)$ and $\bar{K}(k+1) = \bar{P}(k+1|k)C^T(C\bar{P}(k+1|k)C^T + R)^{-1}$. In the ub-KF, $\mathbb{E}[A(Z)A(Z)^T]$ can be computed in advance but we need to compute $\lambda_{\max}$ at each step of the algorithm. But if the size of $\mathcal{Z}$ is large, it is more efficient than the modified KF. (Notice that the computation of $\lambda_{\max}$ requires a polynomial number of operations in N while the size of $\mathcal{Z}$ can be exponential in N.)

Lemma 3: If $\bar{P}(k|k) \succeq P(k|k)$, then $\bar{P}(k+1|k) \succeq P(k+1|k)$.

Proof: Let $M = \hat{x}(k|k)\hat{x}(k|k)^T$ and I be an identity matrix. Then using (9),

$$\begin{aligned} &\bar{P}(k|k) - P(k|k) \\ &= \lambda_{\max}\mathbb{E}[A(Z)A(Z)^T] \\ &\quad -\mathbb{E}[A(Z)P(k|k)A(Z)^T] - \mathbb{E}[A(Z)MA(Z)^T] \\ &= \mathbb{E}[A(Z)(\lambda_{\max}(\bar{P}(k|k))I - P(k|k))A(Z)^T] \\ &\quad +\mathbb{E}[A(Z)(\lambda_{\max}(M)I - M)A(Z)^T]. \end{aligned}$$

Since $\bar{P}(k|k) \succeq P(k|k)$ and $\lambda_{\max}(S)I - S \succeq 0$ for any symmetric matrix S, $\bar{P}(k|k) - P(k|k) \succeq 0$. ■

Using Lemma 3, Lemma 2, and the induction hypothesis, we obtain the following theorem. The ub-KF maintains the state error covariance which is lower-bounded by the state error covariance of the modified KF.

Theorem 2: If the ub-KF starts with an initial covariance $\bar{P}(0|0)$, such that $\bar{P}(0|0) \succeq P(0|0)$, then $\bar{P}(k|k) \succeq P(k|k)$ for all $k \geq 0$.

V. Convergence

In this section, we discuss conditions for stable state estimation of the modified KF for DNCSs. Such condition is studied in [4], in which there is a lossy communication channel between the plant and the estimator, and the result is extend to the case with two communication links in [5]. However, the extension of their results to the general case with an arbitrary number of communication links is unknown. While computing the exact communication link probabilities required for stable state estimation is nontrivial, the general conditions for stable state estimation can be found using jump linear system theory.

Definition 1: A DNCS model (6) is mean square stable (MSS) if, for any initial condition x_0 and second-order independent wide sense stationary random process $\{w(k)\}$, there exist x^* and P^* independent of x_0 such that:

(a) $\| \mathbb{E}[x(k)] - x^* \| \to 0$ as $k \to \infty$
(b) $\| \mathbb{E}[x(k)x(k)^T] - P^* \| \to 0$ as $k \to \infty$.

For a discrete-time Markov jump linear system, there is a stationary filter with a finite state error covariance if the system is mean square stable (MSS) and the governing Markov chain is ergodic (Theorem 2 of [19]). Since the communication configuration is independent over time, if a DNCS is MSS, then the state error covariance of the modified KF converges. We can use the following conditions to check if a DNCS is MSS. In addition, using the optimization techniques developed in [17], we can find the range of communication link probabilities for mean square stability.

Theorem 3 (Corollary 1 of [18]): The DNCS model (6) is MSS if and only if there exists $G \succ 0$ such that

$$G - \sum_{z=\mathcal{Z}} p_z A(z)^T G A(z) \succ 0.$$

Theorem 4 (Theorem 2 of [17]): The DNCS model (6) is MSS if

$$\sum_{z\in\mathcal{Z}} p_z \rho(A(z)^T A(z)) < 1,$$

where $\rho(A)$ denotes the spectral radius of A.

The following theorem shows a simple condition under which the state error covariance can be unbounded.

Theorem 5: If $(\mathbb{E}[A(Z)]^T, \mathbb{E}[A(Z)]^T C^T)$ is not stabilizable, or equivalently, $(\mathbb{E}[A(Z)], C\mathbb{E}[A(Z)])$ is not detectable, then there exists an initial covariance $P(0|0)$ such that $P(k|k)$ diverges as $k \to \infty$.

Proof: Let us consider the lb-KF. Let $\underline{P}_k = \underline{P}(k|k)$, $\psi = GQG^T$, $\hat{A} = \mathbb{E}[A]$, and

$$F = -(C\hat{A}\underline{P}_k\hat{A}^T C^T + C\psi C^T + R)^{-1}(C\psi + C\hat{A}\underline{P}_k\hat{A}^T).$$

Then, based on the Riccati difference equation [20], we can express $\underline{P}_{k+1}$ as

$$\begin{aligned}
\underline{P}_{k+1} &= \hat{A}\underline{P}_k\hat{A}^T + \psi \\
&- F^T\left(C\hat{A}\underline{P}_k\hat{A}^T C^T + C\psi C^T + R\right)F \\
&= (\hat{A}^T + \hat{A}^T C^T F)^T \underline{P}_k (\hat{A}^T + \hat{A}^T C^T F) \\
&+ F^T(C\psi C^T + R)F + \psi C^T F + F^T C\psi + \psi.
\end{aligned}$$

Hence, if $(\hat{A}^T + \hat{A}^T C^T F)$ is not a stability matrix, for some $\underline{P}_0 \preceq P(0|0)$, $\underline{P}_k$ diverges as $k \to \infty$. Since the state error covariance of the lb-KF diverges and $\underline{P}(k|k) \preceq P(k|k)$ for all $k \geq 0$ (Theorem 1), $P(k|k)$ diverges as $k \to \infty$. ■

VI. Simulation Results

In simulation, we study the performance of the modified Kalman filtering algorithm shown in Section III against the standard Kalman filter which assumes no communication errors. Then we provide motivating examples showing the effectiveness of the lb-KF and ub-KF.

Our simulation is based on a scenario inspired by the model by Vicsek *et al.* [13]. Consider a general DNCS system (6) consisting of $N = 5$ agents. The state vector of each agent is $x = [x, y, \dot{x}, \dot{y}]^T$, where (x, y) and $(\dot{x}, \dot{y})$ are the position and the velocity components of the vehicle along the x and y axes, respectively.

The agent 1 is a leader and its dynamics is modeled as

$$x_1(k+1) = A_{11}x_1(k) + B_1 u_1(k) + G_1 w_1(k),$$

where $u_1(k) \in \mathbb{R}^{n_u}$ is a control input to the leader agent and $B_1 \in \mathbb{R}^{n_x \times n_u}$.

The dynamics of an agent $i > 0$ is

$$x_i(k+1) = \sum_{j=i-1}^{i+1} A_{\kappa(j)i}(Z)x_{\kappa(j)}(k) + G_i w_i(k),$$

where $\kappa(j) = (j-1 \mod N) + 1$. For $\kappa(j) = i$,

$$A_{ii}(Z) = \begin{bmatrix} 1 & 0 & \delta & 0 \\ 0 & 1 & 0 & \delta \\ 0 & 0 & \frac{1}{S(i)} & 0 \\ 0 & 0 & 0 & \frac{1}{S(i)} \end{bmatrix} \quad G_i = \begin{bmatrix} \frac{\delta}{2} & 0 \\ 0 & \frac{\delta}{2} \\ \delta & 0 \\ 0 & \delta \end{bmatrix},$$

where δ is the sampling interval. For $\kappa(j) \neq i$,

$$A_{\kappa(j)i}(Z) = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & Z_{\kappa(j)i}/S(i) & 0 \\ 0 & 0 & 0 & Z_{\kappa(j)i}/S(i) \end{bmatrix}$$

with $S(i) = \sum_{j=i-1}^{i+1} Z_{\kappa(j)i}$. Hence, when the agent i communicates with its neighboring agents $\kappa(i-1)$ and $\kappa(i+1)$, its new velocity is the average of its velocity and velocities received from its neighboring agents. In addition, $\delta = 1$ and $Q_i = \text{diag}(0.01^2, 0.01^2)$

The mission of this multi-agent system is to visit sites of interests in minimum time with a bounded control input. The mission scenario is shown in Figure 1 along with the trajectory of the leader agent. The control inputs to the

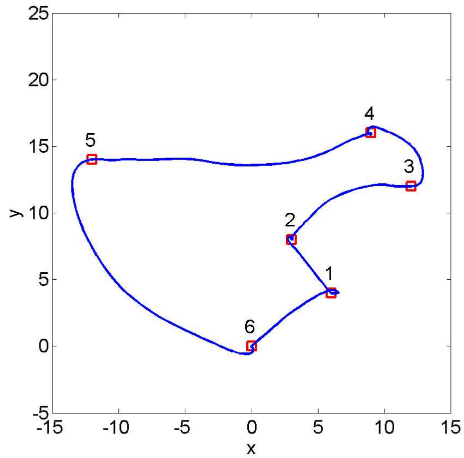

Fig. 1. The mission of the multi-agent system is to visit sites of interests (shown in squares) sequentially from site 1 to site 6 (starting from site 6). The trajectory of the leader agent is shown in solid line. The control inputs to the leader are computed using the robust minimum-time control described in [2].

leader are computed using the robust minimum-time control described in [2]. The trajectories of all agents at different times are shown in Figure 3.

We first study the performance gap between the modified KF against the standard KF which does not assume communication losses. Let the communication matrix be

$$P_{\text{com}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ \lambda & 1 & \lambda & 0 & 0 \\ 0 & \lambda & 1 & \lambda & 0 \\ 0 & 0 & \lambda & 1 & \lambda \\ \lambda & 0 & 0 & \lambda & 1 \end{bmatrix}. \quad (16)$$

The measurement model (7) is used where C is a 10×20 matrix such that $y(k)$ consists of noisy position measurements of all agents and $R = \text{diag}(0.1^2, \ldots, 0.1^2)$. We varied λ from 0.1 to 1.0 with a 0.1 increment. For each value of λ, 100 test cases are generated. For each test case, we ran the modified KF and the standard KF and computed the mean square error (MSE) of state estimates. The result is shown in Figure 2. The figure shows a clear benefit of the modified KF when the communication loss uncertainty is higher. In addition, the modified KF shows an excellent performance for all values of λ.

We now consider two cases: *Case A* and *Case B*. *Case A* is the model described above with $\lambda = 0.7$. *Case B* is the same as *Case A* except C is a 6×20 matrix such that $y(k)$ consists of noisy position measurements of agent 1, 3, and 4. The positions of agents 2 and 5 are not observed in *Case B*. The results are summarized in Table I. The modified KF performs well compared to the standard KF but it requires more computation time. The approximate KFs perform better than the standard KF without much overhead in run-time. Since a less number of states are observed in *Case B*, the state uncertainty is higher in *Case B* and the ub-KF performs better than the lb-KF for *Case B*.

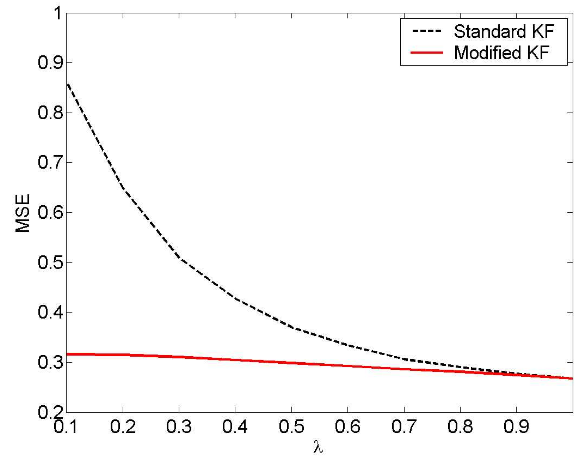

Fig. 2. The average MSE as a function of λ in (16). For each value of λ, 100 test cases are used to compute the average MSE. As the value of λ decreases, the performance gap between the modified KF and the standard KF increases.

TABLE I

COMPARISON OF DIFFERENT KALMAN FILTERS: STANDARD KF, MODIFIED KF (MOD-KF), LOWER-BOUND KF, AND UPPER-BOUND KF

		KF	mod-KF	lb-KF	ub-KF
Case A	MSE	0.303	0.283	0.292	0.352
	Run-time	0.56s	11.69s	0.64s	0.81s
Case B	MSE	0.696	0.542	0.748	0.512
	Run-time	0.52s	11.69s	0.60s	0.87s

VII. CONCLUSIONS

In this paper, we have described efficient approximate filtering algorithms for estimating states of a distributed networked control system (DNCS). A DNCS is an extension of an NCS to model a distributed multi-agent system such as the Vicsek model, where multiple agents communicate over a lossy communication channel. While the time complexity of the exact estimation method can be exponential in the number of communication links, the time complexity of an approximate method is not dependent on the number of communication links. We have also described the general conditions for stable state estimation using jump linear system theory.

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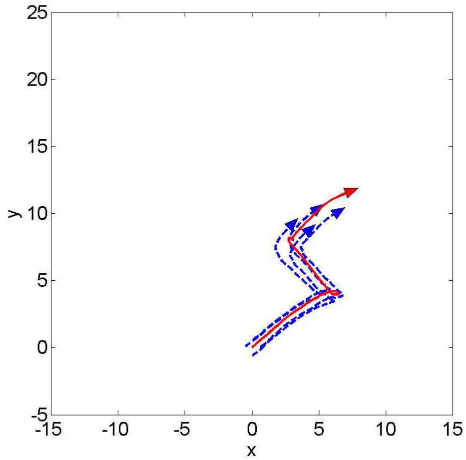
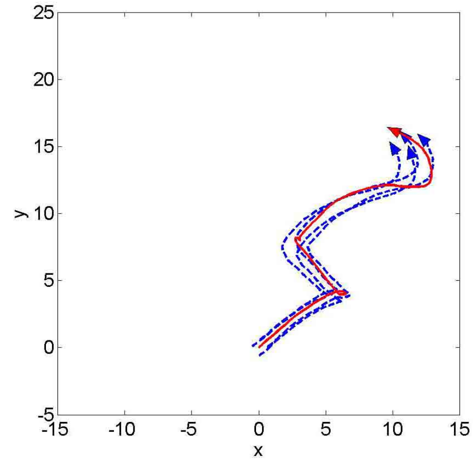
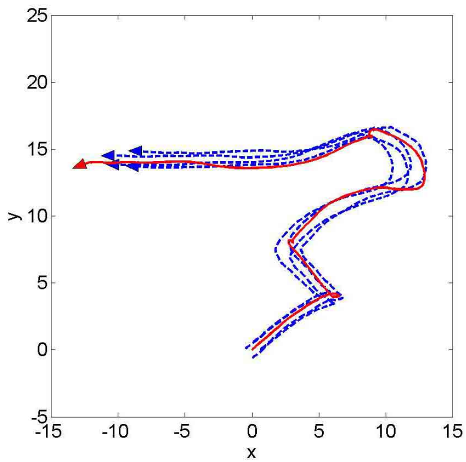


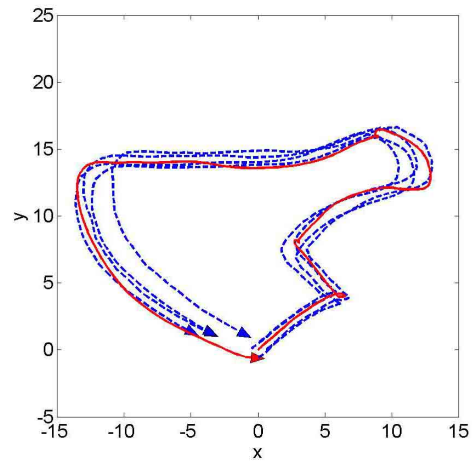
(a) $k = 100$



(b) $k = 140$



(c) $k = 220$



(d) $k = 270$

Fig. 3. Trajectories of all agents at different times (leader in a solid line, followers in dashed lines).

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